Quick Report about the EPFC Contest 15

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Le lâcher de boules de pétanque

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All source codes and LaTeX sources are available on Bitbucket:
https://bitbucket.org/OPiMedia/epfc-contest-15-2019-le-lacher-de-boules-de-petanque

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1 Examples of result

N balls	1	2	2	5
E levels	5	5	100	1000
$L_{N,E}$ launches	5	3	14	11

2 Function corresponding to the problem

$$\forall N \in \mathbb{N}^*, \forall E \in \mathbb{N} : \left| \begin{array}{ll} L_{1,E} & := E \\ L_{N,0} & := 0 \\ L_{N,E} & := 1 + \min\limits_{1 \leq i \leq E} \max\{L_{N-1,i-1}, L_{N,E-i}\} \qquad \forall N \geq 2, \forall E \geq 1 \end{array} \right.$$

E
$\dots$
i
$\dots$
1

The recursive case adds 1 launch to the minimum result on all recursive possible levels to launch. For each launch on the level i, when the ball is broken there is one less ball and we recursive compute on the $i-1$ bottom levels. And when the ball is not broken we recursive compute on the $E-i$ top levels.

$$\begin{array}{rll} \min \{ \max\{L_{N-1,0}, & L_{N,E-1}\}, \\ \rotatebox{90}{$\leq$} & \rotatebox{90}{$\geq$} \\ \max\{L_{N-1,1}, & L_{N,E-2}\}, \\ \rotatebox{90}{$\leq$} & \rotatebox{90}{$\geq$} \\ \max\{L_{N-1,2}, & L_{N,E-3}\}, \\ \rotatebox{90}{$\leq$} & \rotatebox{90}{$\geq$} \\ \dots & \\ \rotatebox{90}{$\leq$} & \rotatebox{90}{$\geq$} \\ \max\{L_{N-1,E-2}, & L_{N,1}\}, \\ \rotatebox{90}{$\leq$} & \rotatebox{90}{$\geq$} \\ \max\{L_{N-1,E-1}, & L_{N,0}\}\} \end{array}$$

3 First cases

$$
\begin{array}{|llll}
L_{N,0} &= 0 & & \\
L_{N,1} &= 1 & & \\
L_{N,2} &= 2 & & \\
L_{N,3} &= 2 \quad \forall N \geq 2, & L_{1,3} = 3 & \\
L_{N,4} &= 3 \quad \forall N \geq 2, & L_{1,4} = 4 & \\
L_{N,5} &= 3 \quad \forall N \geq 2, & L_{1,5} = 5 & \\
L_{N,6} &= 3 \quad \forall N \geq 2, & L_{1,6} = 6 & \\
L_{N,7} &= 3 \quad \forall N \geq 3, & L_{1,7} = 7, & L_{2,7} = 4 \\
L_{N,8} &= 4 \quad \forall N \geq 2, & L_{1,8} = 8 & \\
L_{N,9} &= 4 \quad \forall N \geq 2, & L_{1,9} = 9 & \\
L_{N,10} &= 4 \quad \forall N \geq 2, & L_{1,10} = 10 & \\
L_{N,11} &= 4 \quad \forall N \geq 3, & L_{1,11} = 11, & L_{2,11} = 5
\end{array}
$$

4 Bounds

Property 1 $0 \leq L_{N,E} \leq E$

Property 2
$$
\begin{array}{|l}
E = 0 \iff L_{N,E} = 0 \\
E = 1 \iff L_{N,E} = 1
\end{array}
$$

5 Inequalities with fixed number of balls

Property 3 (One more level) $L_{N,E+1} \geq L_{N,E}$

***Conjecture* 1 (Increasing)** [1] $L_{N,E+1} = L_{N,E}$ or $1 + L_{N,E}$

***Conjecture* 2** $\forall k \in \mathbb{N} : L_{N,E} \leq L_{N,E+k} \leq k + L_{N,E}$

***Conjecture* 3 (Fixed points)** $(N = 1 \text{ or } E \in \{0, 1, 2\}) \iff L_{N,E} = E$

Property 4 (Covariant)
$$
\begin{array}{|l}
E' > E \Longrightarrow L_{N,E'} \geq L_{N,E} \\
L_{N,E'} > L_{N,E} \Longrightarrow E' > E
\end{array}
$$

[1] This result and following conjectures seem to be true but I didn't *proved* them. Maybe they are obvious, maybe not.

6 Inequalities with fixed number of levels

Property 5 (One more ball) $L_{N+1,E} \leq L_{N,E}$

Property 6 (Contravariant) $\left| \begin{array}{l} N' > N \Longrightarrow L_{N',E} \leq L_{N,E} \\ L_{N',E} < L_{N,E} \Longrightarrow N' > N \end{array} \right.$

Property 7 (Too more balls) $\forall N \geq E \geq 1 : L_{E,E} = L_{N,E}$

Property 8 (Lower bound) $\forall E \geq 1 : L_{E,E} \leq L_{N,E}$

7 Inequalities

Property 9 (One more ball and one more level) $L_{N+1,E+1} \leq 1 + L_{N,E}$

Property 10 (Biggest choice in recursive calls)
$\forall N \geq 2, \forall i \in \mathbb{N}^* : \left(\lceil \frac{E+1}{2} \rceil \leq i \leq E \Longrightarrow L_{N-1,i-1} \geq L_{N,E-i} \right)$

Proof. $L_{N-1,i-1} \geq L_{N,i-1}$ and $i - 1 \geq E - i \Longrightarrow L_{N,i-1} \geq L_{N,E-i}$ □

8 Simplification

Property 11 (Lighter recursive formulation) $\forall N \geq 2, \forall E \geq 1 :$

$$L_{N,E} = 1 + \min \left\{ \min_{1 \leq i \leq \lfloor \frac{E}{2} \rfloor} \max\{L_{N-1,i-1}, L_{N,E-i}\},\ L_{N-1,\lfloor \frac{E}{2} \rfloor} \right\}$$

***Conjecture* 4 (Logarithmic reduction)** $\begin{array}{l} \forall k \in \mathbb{N} : L_{N,2^k} = L_{\min\{N,k+1\},2^k} \\ \forall E \geq 1 : L_{N,E} = L_{\min\{N,\lfloor \lg(E) \rfloor+1\},E} \end{array}$

9 Table

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
2			2	2	3	3	3	4	4	4	4	5	5	5	5	5	6	6	6	6	6	6	7	7	7	7	7	7	7	8	8	8	8
3				2	3	3	3	3	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	5	6	6	6	6	6	6	6
4					3	3	3	3	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6	6
5						3	3	3	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6
6							3	3	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6
7								3	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6
8									4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6
9										4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6
10											4	4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6
11												4	4	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6
12													4	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6
13														4	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6
14															4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6
15																4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6
16																	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6
17																		5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6
18																			5	5	5	5	5	5	5	5	5	5	5	5	5	5	6
19																				5	5	5	5	5	5	5	5	5	5	5	5	5	6
20																					5	5	5	5	5	5	5	5	5	5	5	5	6
21																						5	5	5	5	5	5	5	5	5	5	5	6
22																							5	5	5	5	5	5	5	5	5	5	6
23																								5	5	5	5	5	5	5	5	5	6
24																									5	5	5	5	5	5	5	5	6
25																										5	5	5	5	5	5	5	6
26																											5	5	5	5	5	5	6
27																												5	5	5	5	5	6
28																													5	5	5	5	6
29																														5	5	5	6
30																															5	5	6
31																																5	6
32																																	6

10 Function lengths

$\forall N \in \mathbb{N}^*, \forall L \in \mathbb{N}$: $l_{N,L}$:= number of appearances of the number of launches L for the number of balls N and all number of levels

Property 12 $l_{N,0} = 1$

***Conjecture* 5 (Recursive formulation)** $\forall N \geq 2, \forall L \geq 1 : l_{N,L} = l_{N-1,L-1} + l_{N,L-1}$

***Conjecture* 6** $\begin{cases} l_{1,L} = 1 \\ l_{2,L} = L, \quad \forall L \geq 1 \\ l_{3,L} = 1 + \frac{L(L+1)}{2} & \text{OEIS A000124 shifted} \\ l_{4,L} & \text{OEIS A000125 shifted} \\ l_{5,L} & \text{OEIS A000127 shifted} \\ l_{6,L} & \text{OEIS A006261 shifted} \end{cases}$

11 Function sum of the first lengths

$\forall N \in \mathbb{N}^*, \forall L \in \mathbb{N} : S_{N,L} := \sum_{1 \leq i \leq L} l_{N,i}$

Property 13 $S_{N,0} = 0$

***Conjecture* 7** $\begin{cases} S_{1,L} = L \\ S_{2,L} = \frac{L(L+1)}{2} & \text{triangular numbers} & \text{OEIS A000217} \\ S_{3,L} = \frac{L(L^2+5)}{3!} & & \text{OEIS A004006} \\ S_{4,L} = \binom{L+1}{4} + \binom{L+1}{2} = \frac{L(L+1)[(L-2)(L-1)+3\times4]}{4!} & & \text{OEIS A055795 shifted} \\ S_{5,L} & & \text{OEIS A057703} \\ S_{6,L} & & \text{OEIS A115567} \end{cases}$

***Conjecture* 8** $S_{n,L} \in \mathcal{O}\left(\frac{L^n}{n!}\right) \qquad \frac{L^n}{n!} \simeq \frac{1}{\sqrt{2\pi n}}\left(\frac{eL}{n}\right)^n \qquad S_{n,L} \in \mathcal{O}\left(\frac{L^n}{n!}\right)$

12 New way to solve the problem

***Conjecture* 9** $L_{2,E} = \lfloor \sqrt{2E} + 1/2 \rfloor$ see OEIS A123578 and OEIS A002024

***Conjecture* 10** $L_{N,E} = \min\{L \in \mathbb{N} \mid S_{N,L} \geq L\}$