

Disjoint Compatible Perfect Matchings

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- 1** Some basic definitions
- 2 Sketch of proof
- 3 Web page
- 4 References



Planar straight line graph (PSLG)

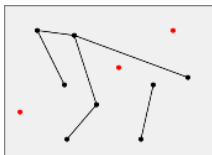
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A *planar straight line graph*

- is an undirected graph
- each vertex is a point in the plane
- each edge is a segment between two points (no curve)
- no segment intersection



Perfect matching

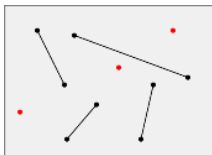
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A *matching* is a set of segments with no point in common (each vertex has degree at most one)



A matching is *perfect* if and only if each vertex has degree one



Canonical perfect matching

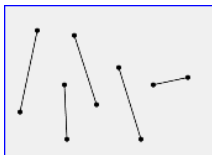
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- S a set of $2n$ points
- $p_1, p_2, p_3, \dots, p_{2n}$ in increasing order of their x -coordinates (and if necessary of their y -coordinates),

The *canonical perfect matching* of S , writed $N(S)$, is the perfect matching with segments

$$p_1 - p_2, p_3 - p_4, p_5 - p_6, \dots, p_{2n-1} - p_{2n}.$$



Compatible perfect matchings

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Consider now two perfect matchings.

Two perfect matchings are *compatible* if and only if their union is with no intersection.

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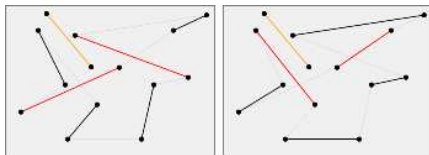


Figure: These two perfect matchings are **not** compatible.

Be careful, the union is the union of two sets (concept of set theory).
The intersection is the intersection of two segments (geometrical concept).



Transformation between two perfect matchings

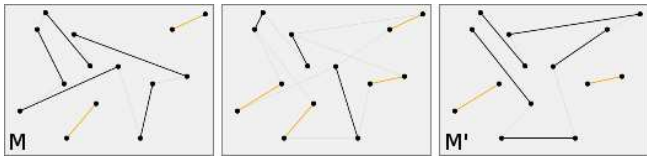


Figure: Transformation of length 2

- S a set of points
- M and M' two perfect matchings of S
- a *transformation* between M and M' of length k is a sequence $M = M_0, M_1, M_2, \dots, M_k = M'$ of perfect matchings of S
- such that $\forall i : M_i$ and M_{i+1} are compatible

Theorem

\forall perfect matchings M and M' ,
 \exists transformation of length at most $2\lceil \lg(n) \rceil$ between M and M'



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Lemma i

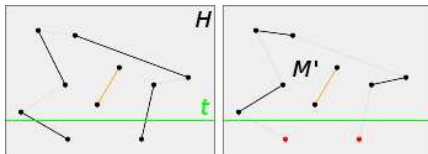
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Lemma

\forall perfect matching M ,

\forall line t cutting an even number of segments of M (t contains no vertex),
let H the halfplane determined by t ,

let S the set of vertices of M in H ,

\exists perfect matching M' of S : M and M' are compatible



Lemma ii

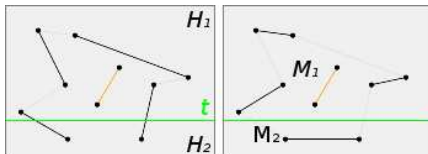
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Lemma

\forall perfect matching M ,

\forall line t cutting an even number of segments of M (t contains no vertex),

let halfplanes H_1 and H_2 determined by t ,

let S_1 and S_2 sets of vertices of M in H_1 and in H_2 ,

\exists perfect matchings M_1 of S_1 and M_2 of S_2 : M and $(M_1 \cup M_2)$ are compatible

Proof.

- by lemma i, \exists perfect matchings M_1 of S_1 and M_2 of S_2 :
 M and M_1 are compatible, and M and M_2 are compatible
- M_1 and M_2 are separated,
thus $M_1 \cup M_2$ is a perfect matching compatible with M





Lemma iii

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Lemma

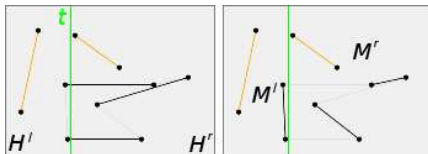
$\forall S$ of $2n$ points,

\forall perfect matchings M of S ,

\exists transformation of length at most $\lceil \lg(n) \rceil$ between M and $N(S)$

Proof.

With S set of $2n$ points, proof by induction on n .



- Cut the plane in two and apply lemma ii on each half.
- Union of transformation of each parts.





Theorem

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Theorem

\forall perfect matchings M and M' ,
 \exists transformation of length at most $2\lceil\lg(n)\rceil$ between M and M'

Proof.

S the set of $2n$ points.

By lemma iii, \exists perfect matchings M and M' :

$$M = M_0, M_1, M_2, \dots, M_k = N(S) \text{ and}$$

$$M' = M'_0, M'_1, M'_2, \dots, M'_{k'} = N(S) \text{ with } k, k' \leq \lceil\lg(n)\rceil.$$

Thus $M_0, M_1, M_2, \dots, M_k = M'_k, \dots, M'_2, M'_1, M'_0 = M'$ is a transformation of length at most $2\lceil\lg(n)\rceil$. □



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Web page and demonstration of the application

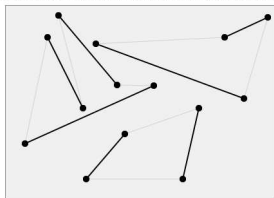
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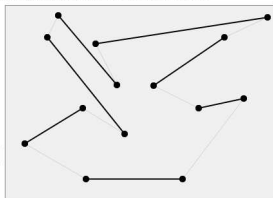
3 Experiment by yourself

3.1 Interactive application

Click in the two zones to add/remove point or segment (only one segment by point and with no intersection). You can also click on one matching example to load it in the interactive zones. Wait a moment above a button to bring up a short explanation tooltip.



7 segments Even? Canonical Shuffle
Perfect? Canonical? Vertical-horizontal?



7 segments Even? Canonical Shuffle
Perfect? Canonical? Vertical-horizontal?

14 points ? possible perfect matching(s) ≤ 135135 Minimal length transformation ≤ 8
Different? Disjoint? (common segments)
Compatible? (intersect segments)

- Add/remove point
- Add/remove segment: vertical-horizontal

- Draw only segments of the matching
- Draw also segments of consecutive matchings:
- Draw also segments of all other matchings:

Clear Load as... Parcourir... Aucun fichier sélectionné. Save to "matchings.txt"

4 matchings: Perfect | Disjoint | Compatible | Disjoint compatible | Transformation | Disjoint | Clear list





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Thank you!

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Questions time...