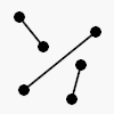

Disjoint Compatible Perfect Matchings

Olivier Pirson

INFO-F420 *Computational geometry*

ULB

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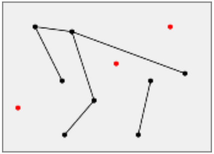
(Some corrections November 26, 2017)

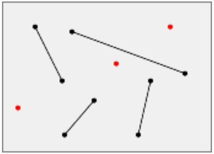
Planar straight line graph (PSLG)



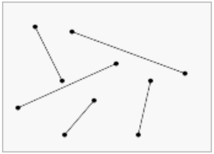
A *planar straight line graph*

- is an undirected graph
- each vertex is a point in the plane
- each edge is a segment between two points (no curve)
- no segment intersection

Perfect matching

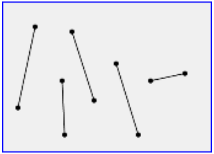


A *matching* is a set of segments with no point in common (each vertex has degree at most one)



A matching if *perfect* if and only if each vertex has degree one

Canonical perfect matching



- S a set of $2n$ points
- $p_1, p_2, p_3, \ldots, p_{2n}$ in increasing order of their x-coordinates (and if necessary of their y-coordinates),

The *canonical perfect matching* of S, writed $N(S)$, is the perfect matching with segments
$p_1 — p_2, p_3 — p_4, p_5 — p_6, \cdots p_{2n-1} — p_{2n}$.

Compatible perfect matchings

Consider now two perfect matchings.

Two perfect matchings are *compatible*
if and only if their union is with no intersection.

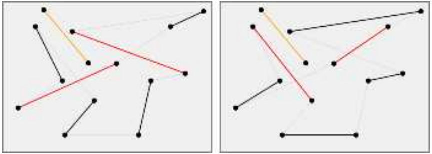


Figure: These two perfect matchings are **not** compatible.

Be careful, the union is the union of two sets (concept of set theory).
The intersection is the intersection of two segments (geometrical concept).

Transformation between two perfect matchings

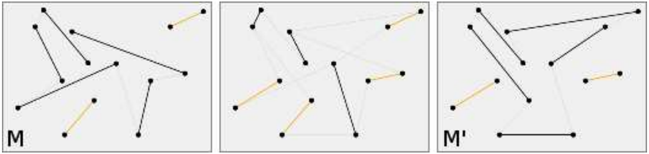
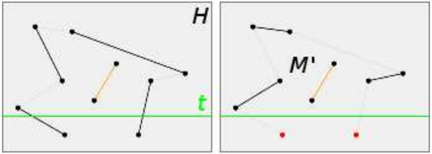


Figure: Transformation of length 2

- S a set of points
- M and M' two perfect matchings of S
- a *transformation* between M and M' of length k is a sequence $M = M_0, M_1, M_2, \dots, M_k = M'$ of perfect matchings of S
- such that $\forall i : M_i$ and M_{i+1} are compatible

Theorem

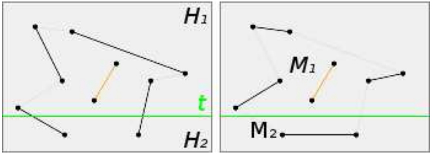
$\forall$ *perfect matchings* M *and* M',
$\exists$ *transformation of length at most* $2\lceil \lg(n) \rceil$ *between* M *and* M'



Lemma

∀ *perfect matching* M*,*
∀ *line* t *cutting an even number of segments of* M *(*t *contains no vertex),*
let H *the halfplane determined by* t*,*
let S *the set of vertices of* M *in* H*,*
∃ *perfect matching* M' *of* S : M *and* M' *are compatible*

Lemma ii



Lemma

∀ *perfect matching* M,
∀ *line* t *cutting an even number of segments of* M *(*t *contains no vertex),*
let halfplanes H_1 *and* H_2 *determined by* t,
let S_1 *and* S_2 *sets of vertices of* M *in* H_1 *and in* H_2,
∃ *perfect matchings* M_1 *of* S_1 *and* M_2 *of* S_2 : M *and* $(M_1 \cup M_2)$ *are compatible*

Proof.

- by lemma i, ∃ perfect matchings M_1 of S_1 and M_2 of S_2 : M and M_1 are compatible, and M and M_2 are compatible
- M_1 and M_2 are separated, thus $M_1 \cup M_2$ is a perfect matching compatible with M

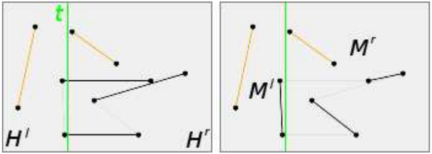
□

Lemma

$\forall S$ of $2n$ points,
$\forall$ perfect matchings M of S,
$\exists$ transformation of length at most $\lceil \lg(n) \rceil$ between M and $N(S)$

Proof.

With S set of $2n$ points, proof by induction on n.



- Cut the plane in two and apply lemma ii on each half.
- Union of transformation of each parts.

□

Theorem

$\forall$ *perfect matchings* M *and* M'*,*
$\exists$ *transformation of length at most* $2\lceil \lg(n) \rceil$ *between* M *and* M'

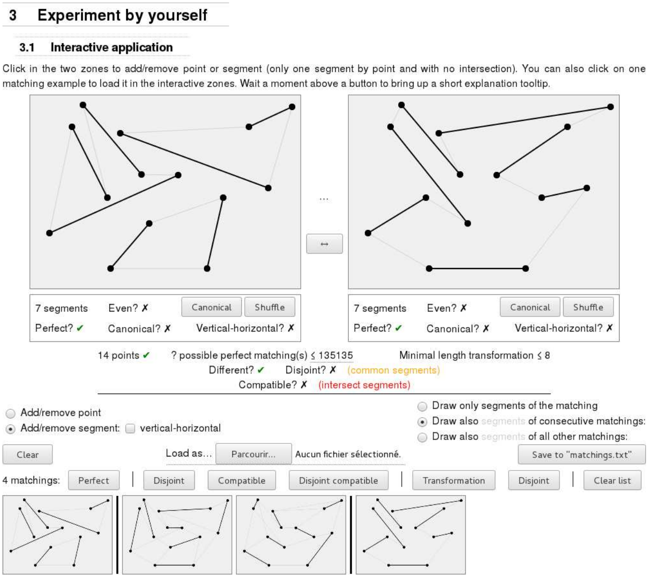
Proof.

S the set of $2n$ points.
By lemma iii, $\exists$ perfect matchings M and M' :

$M = M_0, M_1, M_2, \ldots, M_k = N(S)$ and
$M' = M'_0, M'_1, M'_2, \ldots, M'_{k'} = N(S)$ with $k, k' \leq \lceil \lg(n) \rceil$.

Thus $M_0, M_1, M_2, \ldots, M_k = M'_{k'}, \ldots, M'_2, M'_1, M'_0 = M'$ is a transformation of length at most $2\lceil \lg(n) \rceil$. □

Web page and demonstration of the application



References

Thank you!

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Questions time…